Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - IV

Subject Code : 4TE04EMT1 **Branch: B.Tech (Auto/Civil/EE/EC/Mech)**

Semester : 4 **Date : 15/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The finite Fourier cosine transform of $f(x)=2x,\ 0<x<4$ is

(A) $\frac{32}{n^2\pi^2}\left[(-1)^n-1\right]$ (B) $\frac{16}{n^2\pi^2}\left[(-1)^n-1\right]$ (C) $\frac{32}{n^2\pi^2}(-1)^n$

(D) none of these

b) The Fourier sine transform of $f(x)=\begin{cases}k, & 0<x<a\\ 0, & x>a\end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}k\left(\frac{1-\cos a\lambda}{\lambda}\right)$ (C) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{a}\right)$

(D) none of these

c) Under the inverse transformation $w=\frac{1}{z}$ the straight line $ax+by=0$ transform into

(A) circle (B) straight line passing through origin (C) straight line

(D) none of these

d) Which one of the following is an analytic function

(A) $\mathrm{f}(z)=\mathrm{R}iz$ (B) $\mathrm{f}(z)=\mathrm{Im}\,z$ (C) $\mathrm{f}(z)=\bar{z}$ (D) $\mathrm{f}(z)=\sin z$

e) The unit vector tangent to the curve $x=t, y=t^2, z=t^3$ at the point $(-1,1,-1)$ is

(A) $\frac{1}{\sqrt{14}}(i+2j+3k)$ (B) $\frac{1}{\sqrt{14}}(i-2j+3k)$ (C) $\frac{1}{\sqrt{3}}(i+j+k)$

(D) $\frac{1}{\sqrt{3}}(i-j+k)$

f) The value of the line integral $\int\nabla(x+y-z)\cdot d\vec{r}$ from $(0,1,-1)$ to $(1,2,0)$ is

(A) – 1 (B) 3 (C) 0 (D) none of these

g) E^{-1} equal to
(A) $1-\nabla$ (B) $1+\nabla$ (C) $1+\delta$ (D) $1-\delta$

h) hD equal to
(A) $\log(1+\Delta)$ (B) $\log(1-\Delta)$ (C) $\log(1+E)$ (D) $\log(1-E)$

i) The nth difference of a polynomial of degree n is
(A) constant (B) zero (C) $n!$ (D) none of these

j) In application of Simpson's $\frac{1}{3}$ rule, the interval of integration for closer approximation should be
(A) odd and small (B) even and small (C) even and large
(D) none of these

k) The convergence in the Gauss – Seidel method is faster than Gauss – Jacobi method.
(A) TRUE (B) FALSE

l) The Gauss – Jordan method in which the set of equations are transformed into diagonal matrix form.
(A) TRUE (B) FALSE

m) Using modified Euler's method, the value of $y(0.1)$ for $\frac{dy}{dx}=x-y,\ y(0)=1$ is
(A) 0.909 (B) 0.809 (C) 0.0809 (D) 0.0908

n) Which of the following methods is the best for solving initial value problems:
(A) Taylor's series method (B) Euler's method
(C) Runge-Kutta method of 4^{th} order (D) Modified Euler's method

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Use Stirling's formula to find y_{28} given that **(5)**
$y_{20}=49225,\ y_{25}=48316,\ y_{30}=47236,\ y_{35}=45926$ and $y_{40}=44306$.

b) Construct Newton's forward interpolation polynomial to the following data: **(5)**

x	4	6	8	10
y	1	3	8	16

c) Find the Fourier sine transform of $f(x)=\begin{cases}0 & 0<x<a\\ x & a\le x\le b\\ 0 & x>b\end{cases}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations by Gauss-Seidal method. **(5)**
$10x_1+x_2+2x_3=44,\ 2x_1+10x_2+x_3=51,\ x_1+2x_2+10x_3=61$

b) The population of a certain town is shown in the following table: **(5)**

Year	1961	1971	1981	1991	2001
Population (in thousands)	19.96	36.65	58.81	77.21	94.61

Find the rate of growth of population in 1991.

c) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate of $u(x, y)$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Use Euler's method to find an approximate value of y at $x = 0.1$, in five steps, given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$. **(5)**

b) Evaluate $\int_0^{0.6} e^{-x^2}\, dx$ by using Simpson's 1/3rd rule. **(5)**

c) Solve the following system of equations using Gauss-elimination method: **(4)**

$-x_1 + x_2 + 2x_3 = 2,\ 3x_1 - x_2 + x_3 = 6,\ -x_1 + 3x_2 + 4x_3 = 4$

Q-5 **Attempt all questions** **(14)**

a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied. **(5)**

b) Using Green's Theorem, evaluate $\oint_C \left[(y - \sin x)\,dx + \cos x\,dy\right]$ where C is the plane triangle enclosed by the lines $y = 0,\ x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. **(5)**

c) Compute $f(9.2)$ by using Lagrange Interpolation formula from the following data: **(4)**

x	9	9.5	11
y	2.1972	2.2513	2.3979

Q-6 **Attempt all questions** **(14)**

a) Prove that $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + 3xz^2k$ is irrotational and find its scalar potential. **(5)**

b) Find the bilinear transformation which sends the points $z = 0, 1, \infty$ into the points $w = -5, -1, 3$ respectively. What are the invariant points of the transformation? **(5)**

c) Solve $\frac{dy}{dx} = 3 + 2xy$ where $y(0) = 1$ for $x = 0.1$ by Picard's method. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Using Cauchy – Riemann equations, prove that if $f(z) = u + iv$ is analytic with constant modulus, then u, v are constants. **(5)**

b) If $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^3\hat{k}$, show that $\int_C \vec{F} \cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path joining A(1, – 2, 1) to B(3, 1, 4). **(5)**

c) The function $f(x)$ is given as follows: **(4)**

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	1	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

Compute the integral of $f(x)$ between $x = 0$ and $x = 1.0$ using Trapezoidal rule.

Q-8 **Attempt all questions** **(14)**

a) Using Taylor's series method, compute $y(-0.1)$, $y(0.1)$, $y(0.2)$ correct to four decimal places, given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ **(5)**

b) Find the Fourier cosine and sine integral of $f(x) = e^{-kx}$ $(x > 0,\ k > 0)$. **(5)**

c) Find the angle between the tangents to the curve $x = t^2, y = 2t, z = -t^3$ at the points $t = 1$ and $t = -1$. **(4)**